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ON THE EFFECTIVENESS OF VARIOUS MODES OF DETONATION OR COMBUSTION

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INTRODUCTION

The present report originated from a specific question: can the effectiveness of a jet propulsion motor be increased by controlling the mode of detonation or combustion? Methods previously developed for gas dynamical problems by the Applied Mathematics Group of New York University provide a basis for making a theoretical contribution by computing the pressure as a function of the time under highly idealized assumptions, thereby comparing the effectiveness of different modes of reaction. Such computations are presented in the present memorandum.

However, the value of the study should not be judged solely in relation to the original specific problem. Methods and general results may well be useful for many other problems of detonation and combustion.

The work was carried out by Dr. Charles De Prima and Miss Anneli Leopold under the guidance of Dr. K. O. Friedrichs.

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On the Effectiveness of Various Modes of
Detonation or Combustion

1. Problem. In connection with the method of cascading in jetmotors proposed by Dr. F. Zwicky from the Aerojet Co. [1],[2] we have investigated the effectiveness of various modes of detonation or combustion on the basis of a simplified model.

We have assumed that a straight tube, extending to infinity in one direction and closed at the other end, contains explosive gas in a section adjacent to the closed end and air in the remaining part. Then we have considered the following four modes of detonation and two modes of combustion.

DI: The explosive gas is ignited at the closed end in such a way that a detonation wave is produced which sweeps over the section of explosive gas. (See Figure 1).

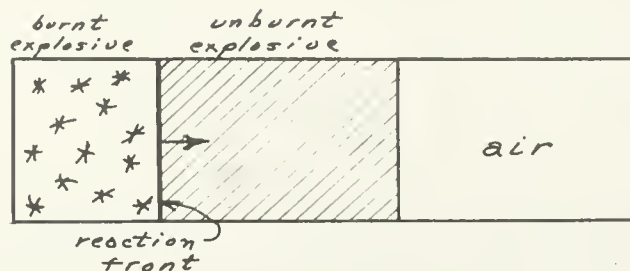


Figure 1

DII: The explosive gas is ignited at the interface between air and explosive in such a way that a detonation wave is produced which moves toward the closed end. (See Figure 2).

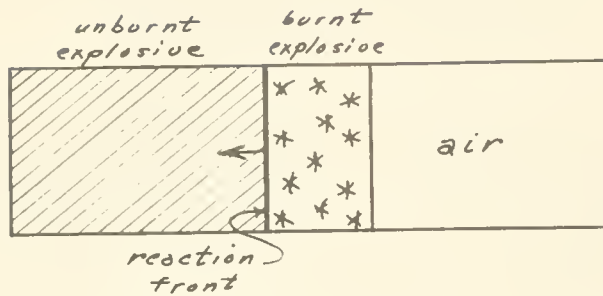


Figure 2

DIII: The whole section of explosive is ignited instantaneously so that a constant volume detonation results.

DIV: The explosive gas is first adiabatically compressed by the action of a piston, then ignited instantaneously. Hereafter the exploded gas is allowed to expand adiabatically but is forced to work against the piston until it has delivered the work it has received during the pre-compression phase; only then the piston is removed and the exploded gas may expand pushing out the air ahead of it. (See Figure 3).

CI: Ignition at the closed end produces an ordinary combustion wave moving away from the closed end. (Fig.1)

CII: Ignition at the interface produces an ordinary combustion wave moving toward the closed end. (Fig. 2)

To measure the effectiveness* of these modes of operation we have calculated the impulse per unit area delivered to the closed end i.e. the integral of the excess pressure with

* We use the somewhat vague term effectiveness instead of efficiency to avoid confusion since the latter term is generally used for the ratio of useful energy output to energy input.

respect to the time,

$$I = I(t) = \int_0^t [p - p_0] dt,$$

p being the pressure exerted by the gas against the closed end and p_0 being the atmospheric pressure.

Our calculations extend only over a short interval of time; but this interval is large enough to cover the decisive part of the process: the sudden rise and the quick drop of the pressure. After the time up to which our calculations extend, the pressure decreases very slowly. In all modes of operation considered we have assumed that initially the same amount of combustible gas is available. When we say that one mode of operation is more effective than another we mean merely that this mode produces a higher impulse than the other during the time interval considered.

The reason for considering impulse rather than energy as a measure of effectiveness is this: For a jet motor which operates in a periodic process, the average thrust over a period may be considered a measure for effectiveness. This average thrust is given by the expression

$$\frac{1}{T} \int_0^T (p - p_0) dt$$

if T is the period. The main contribution to the value of the integral here is due to a short interval of time after the explosion, during which the pressure rises suddenly and then drops again. Thus, when one compares two processes operating with the same period T , one may assume that the more effective process is that one having the higher value of the impulse

$$I = \int_0^t (p - p_0) dt$$

over the first decisive interval of time.

Naturally, for the discussion of the operation of a jet-propelled device, completely different notions of effectiveness*, in particular an over-all ^{efficiency} ~~efficiency~~ may be needed. Such an over-all efficiency depends on various factors not considered here. Our sole intention in the present report is to furnish only some information about one of the factors to be taken into account when such an over-all efficiency is investigated.

In our calculations we have made particular assumptions about the nature of the explosive gas which we shall explain in detail later on. The initial state was assumed at atmospheric pressure and room temperature.

2. Results. The results in general terms are the following:

The impulse resulting from mode DII exceeds, after a short time, that resulting from mode DI and remains greater by approximately a constant value for some time. In this sense it is more effective to let the detonation proceed toward the wall than away from it. However, the difference is not very great.

To express the result numerically in dimensionless form we must introduce a "unit impulse" I_0 ; to this end we introduce the atmospheric pressure p_0 as unit pressure, the length l_0 of the explosive section as unit length, and as unit time the time $t_0 = l_0/c_0$ which a sound wave in atmospheric air would take to travel across the unit length. If, for example, the unit length $l_0 = 34.7$ cm. = 1.14 ft., then the unit time is just one millisecond. The unit impulse I_0 is then the impulse p.u. area which atmospheric pressure would produce in unit time,

$$I_0 = p_0 t_0 = p_0 l_0 / c_0 .$$

We find, for the particular case considered, described later on, that at the time $t = t_0$ the impulse from mode DI has

* For various notions of efficiency in connection with jet-propelled devices see Zwicky [2],[3].

attained the value $I = 2.9I_0$, and that from mode DII the value $I = 5.2I_0$; from there on the values of I increase only slowly and the difference of the two values of the impulse remains approximately constant, at about the value $2I_0$ for some time. More details can be read off from Table 1.

The speed of the detonation waves in the modes DI and DII is the "natural" one determined on the basis of the Chapman-Jouguet hypothesis (see Manual [4] pp. 129-133 and Supplement [5], Lewis and v. Elbe [6] p.235 and Jost [7] p. 157). Other modes of detonation would result from an artificially propagated detonation. The simplest such artificial mode is the constant volume detonation, DIII, initiated by instantaneous ignition. The value of the impulse in this case remains between those of the modes DI and DII for some time. We could not ascertain whether or not it eventually gets ahead of both and increases at a faster rate than those of the other two modes. An investigation of mode DIII is justified mainly because the calculations for its determination are relatively simpler; not because instantaneous detonation might be produced in actual performance. We were interested in particular in finding out to which degree of accuracy the results of any mode of detonation could be replaced by those of a constant volume detonation; and we have found that this can be done with good accuracy (see Tables 1 and 3, representing impulse and pressure as functions of time for the three modes DI, DII, DIII.)

Still a different mode of detonation had been suggested: First a section of the explosive adjacent to the interface is detonated instantaneously; a second section is detonated after it has been compressed by the shock wave sent out from the first detonation. Successively, one section after another is detonated after it has undergone one shock compression after another resulting from the previous detonations. Due to the high pre-compression of the last sections, a high increase of efficiency could be

expected. It seems most likely, however, that the impulse resulting from such a process lies between those resulting from modes DII and DIII.

To obtain the benefit from pre-compression it seems necessary to produce the pre-compression by external means. Therefore, the idealized mode DIV has been investigated.* Various models may be imagined to produce it. In any case, there are the following four preliminary steps in this idealized mode, as indicated in Figure 3. Initially, the explosive

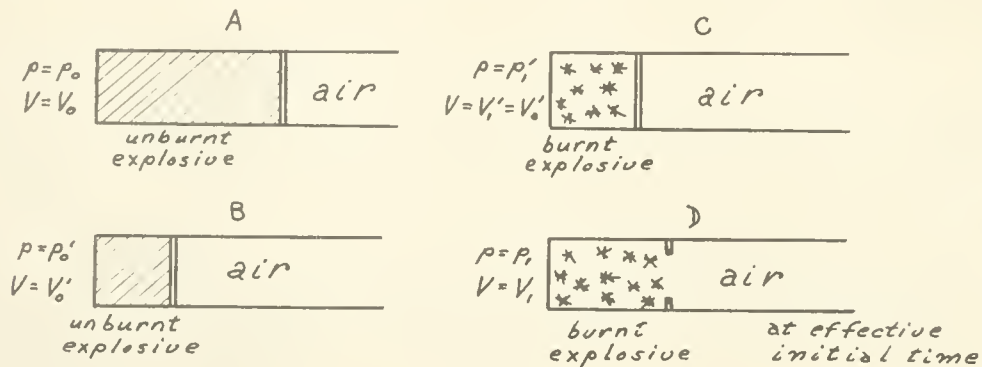


Figure 3

has atmospheric pressure p_0 , and volume V_0 . A piston placed at the interface between explosive and air then compresses the explosive adiabatically to a prescribed volume V_0' and to a pressure p_0' (see Fig. 3, A and B). The piston is to come to rest when this volume is reached. At this instant the explosive is ignited instantaneously, thus acquiring the pressure p_1' but still occupying the volume $V_1' = V_0'$ (see Fig. 3C). The piston then moves backwards and the exploded gas expands, adiabatically by assumption (see Fig. 3D). As soon as the exploded gas has, in pushing back the piston, done as much work as the piston had done in compressing the gas from p_0 to p_0' , the piston is removed.

* It was suggested to us by Mr. E. S. Roberts.

It is easily seen that the total impulse imparted to the tube during this preliminary period of the process is zero. From the time on at which the piston was removed (we shall refer to this time as the "effective initial time"), the exploded gas interacts with the air in the same way as after a constant volume detonation without pre-compression. We have measured the impulse from this effective initial time on. Pressure and volume at this time are denoted by p_1 and V_1 respectively.

This impulse was calculated for various degrees of pre-compression viz. for the ratios

$$V_0/V'_0 = 1.00, 6.03, 103.2$$

of original volume V_0 to pre-compressed volume V'_0 (see Fig. 3, A,B). The resultant impulses are shown in Table 2, the resultant pressures in Table 4. It is seen that with greater pre-compression indeed a greater impulse results; but the increase is not very large. While for case $V_0/V'_0 = 1$, the impulse attains the value $I = 4.75 I_0$ at $t = t_0$ and $I = 7.6 I_0$ at $t = 3t_0$, for $V_0/V'_0 = 6.03$, the impulse is $I = 7.55 I_0$ at $t = t_0$ and $I = 9.7 I_0$ at $t = 3t_0$. The difference thus changed little during this time and remains approximately constant at the value $2.1 I_0$ from then on. It is perhaps surprising that this difference is relatively low, and that the impulse increases only little if the pre-compression is further increased, even to values of the compression ratio as high as $V_0/V'_0 = 103.2$ as seen from Table 4. In this comparison the duration of the preliminary (from the beginning of the pre-compression to the effective initial time) phase has been ignored. This duration depends on the mechanism employed, but one may assume that it will increase with the degree of compression V_0/V'_0 . Therefore, if this duration is long, the impulse will become larger than that from a constant volume detonation only after an accordingly long time.

It was natural to expect that pre-compression would increase the effectiveness considerably since this is the case in a combustion motor. There is, however, a decisive difference between the process in a combustion motor and the process considered here. In a combustion motor, the exploded gas is allowed to move the piston back to the original position, thus performing excess work from the "effective initial time" on. This excess work, which is considerable for a high pre-compression, is used to drive an augmentation device which moves large masses; such as air through an airplane propeller, water through a ship screw, or the "whole earth" by a land vehicle*, and it is only due to this augmentation that the thrust is increased. In the present process, however, in which the piston is removed at the effective initial time, the energy available for the excess work is only used to accelerate and heat the exhaust gases so that no great benefit is derived from the pre-compression. Thus it seems that one should not expect that pre-compression in a jet motor could step up the impulse to the same degree as in a combustion motor unless considerable augmentation is used.

This somewhat negative statement should, however, not be over-rated. Our calculation concerns only the first phase of the operation. The situation is perhaps different if the whole cycle is considered. The following argument might be offered in support of this idea. A rocket motor delivering (at least over a considerable length of time), a steady exhaust flow through a nozzle with atmospheric pressure at the exhaust, produces a thrust given by $F = \dot{m}v$, $\dot{m}$ being the mass flux per unit time and v the exhaust gas velocity; the exhaust flux of kinetic energy is given by $\frac{1}{2} \dot{m}v^2$. Therefore any device which increases the flux of kinetic energy automatically increases the thrust and hence the impulse. For a motor with periodic operation, the impulse over a cycle of period T is $\int_0^T \dot{m}v dt$ and the kinetic energy

* Cf. F. Zwicky, [2], page 31.

ejected during a cycle is $\frac{1}{2} \int_0^T \dot{m} v^2 dt$. It is no longer evident that increase of the ejected kinetic energy implies increase of impulse, but it is still likely to a certain degree. And since a pre-compression device outlined above increases probably the kinetic energy ejected during a cycle, it probably also increases the impulse.

A completely different process results if the ignition does not produce a detonation but only an ordinary combustion process.

We have made the assumption that the width of the zone across which combustion takes place is negligibly small, in other words, that combustion takes place across a discontinuity surface; such combustion processes will also be called "deflagrations." In most actual cases, the combustion zone does not only have a noticeable width, but in general differs considerably from a plane. Frequently, the flow involving combustion is turbulent and not steady. Nevertheless, our assumption can be employed to describe approximately the situation even in such cases (although the approximation is certainly crude) provided that burning takes place in a finite zone whose width remains nearly constant in time and at whose two sides the quantities are almost constant over the cross-section. For, in that case the conservation laws give the same relation for the quantities at both sides of the zone as for both sides of a discontinuity surface. A sufficiently large neighborhood of a curved burning surface or a region of turbulent burning may, to a certain degree, be considered such a zone. (Of course, this argument assumes that losses due to viscous friction and due to heat conduction through the wall can be ignored.)

The decisive difference between the propagation of a plane burning front, of a curved burning front, and of a laminar or turbulent narrow burning zone whose width remains constant in time lies in the speed of propagation. The (normal) burning speed, c_f , of a flame, i.e. the speed of the front relative to the unburnt gas ahead of it depends predominantly on the heat conductivity of

the gases. Its determination involves principles quite different from those determining the natural detonation speed; (cf. Lewis, v. Elbe [6], Jost [7]p.63ff and a forthcoming report of the NYU-AMG). For a plane burning front, the value of the burning speed is very low, of the order of magnitude of .3 to 3 m/sec. For curved burning fronts, the local burningspeed is the same as for a plane front; but when we describe the propagation of a curved flame front approximately by that of a plane burning zone, the "effective" burning speed, c_f , corresponding to such a description is greater. The same is true, to a higher degree, for turbulent burning. (Cf. Jost [7]p. 63ff, 89ff). (Jost mentions in his book on p.61 that flame speeds may be of the order of magnitude of several hundred m/sec). In our calculations we have assumed various values for the effective burning speed, viz.

$$c_f/c_o = .01, = .05, = .15$$

or

$$c_f = 3.47 \text{ m/sec}, = 17.35 \text{ m/sec}, = 52.05 \text{ m/sec}.$$

As indicated in the introduction we have considered two modes of burning: CI, in which the flame begins at the closed end and CII, in which the flame begins at the interface and moves toward the closed end. It is clear that in the case CII the pressure at the closed end starts to build up only after the pre-compression wave sent out by the flame has hit the closed end; therefore the impulse resulting from mode CI will for some time remain greater than that resulting from mode CII; eventually, however, the impulse resulting from mode CII will be greater than that from mode CI. This is seen on Table 1 for $c_f/c_o = .15$. If the effective burning velocity c_f is low, $c_f/c_o = .01$ or $= .05$, the process is so slow that no appreciable impulse will have built up during the time $t = 3t_o$ considered by us. It is most remarkable that for the higher burning speed, $c_f = .15c_o$, the impulse from mode CII assumes the value $I = 5.25I_o$ at $t = 2.4t_o$ and, hence, is larger than that

from the detonation mode DI, at that time. Its value will most likely never exceed that resulting from mode DII; it is at least certain that the rate of increase of the impulse, i.e. the excess pressure, will drop slowly from $t = 1.23t_0$ on and considerably from about $t = 3t_0$ on.

It is of interest to note that the flame speeds U_f in these processes, i.e. the speeds of the flame relative to the tube are much larger than the effective burning speeds. The values, which are different for the two modes CI and CII are indicated in the Table below.

$$c_f/c_0 = \quad .01 \quad \quad .05 \quad \quad .15$$

$$\text{or in m/sec,} \quad c_f = 3.47 \quad , \quad 17.35 \quad , \quad 52.05 \quad .$$

$$\text{CI} \quad \quad U_f/c_0 = \quad .08 \quad \quad .36 \quad \quad .97$$

$$\text{or, in m/sec,} \quad U_f = 27.7 \quad , \quad 125. \quad , \quad 336.$$

$$\text{CII} \quad \quad U_f/c_0 = \quad .04 \quad \quad .22 \quad \quad .70$$

$$\text{or, in m/sec,} \quad U_f = 15.2 \quad , \quad 76.5 \quad , \quad 250.$$

Thus in the case $c_f = .15c_0$ the flame moves with approximately sound speed in mode CI.

3. Properties of the Gases. It was not our intention to assume a specific well-defined explosive and make precise predictions which should be verified experimentally. Our only intention was to obtain typical results. We have assumed the explosive to be an ideal gas whose temperature T is given by

$$T = M^{-1} R \Theta$$

where M is the molecular weight, R the gas constant and Θ the

"reduced temperature"

$$\theta = p\tau = p\rho^{-1} ,$$

p being the pressure, τ the specific volume (volume per unit mass), ρ the density (mass per unit volume). We have further assumed that the internal energy per unit mass, e , depends only on the reduced temperature θ , and not on the pressure; assuming the molecular weight to be constant, or to depend only on θ_0 , this means that the internal energy depends only on the temperature. Denoting the energy of formation per unit mass (at absolute zero temperature) by g we have for the total energy per unit mass

$$E = e + g .$$

For air we simply set $g = 0$ and

$$e = \frac{1}{\gamma_a - 1} \theta , \quad \gamma_a = 1.4 .$$

For the initial temperature $T_0 = 300^\circ\text{K}$ we have $\theta_0 = .0206 \text{ kcal/gm.}$ For the unburnt gas we have assumed (setting $g_u = 0$)

$$E = \frac{1}{\gamma_u - 1} \theta$$

with $\gamma_u = 1.43$. (We have chosen $\gamma_u = 1.43$ instead of 1.4 because the unburnt gas is a mixture of air with a small fraction of oil droplets which affect the weight somewhat but not the volume).

For the burnt gas we have adopted the relation

$$E = - .679 + 2.5\theta + 4.56\theta^2 + 19.86\theta^3 - 36.82\theta^4$$

where θ and E refer to kcal/gm as units. The energy of formation of the burnt gas (at the absolute temperature zero) is

$$g_b = - .679 .$$

Therefore, the "liberated energy" is

$$- g_b = .679 .$$

The ratio of specific heats is to be found from

$$\frac{1}{\gamma-1} = \frac{dE}{d\theta} = 2.5 + 9.12\theta + 59.58\theta^2 - 147.28\theta^3 .$$

For $T = 300^\circ$, 900° , and 3000° K we have $\gamma = 1.4$, 1.31 , and 1.18 , respectively, see Table 6 where E and γ are plotted as functions of θ or T . In the relationship between the temperature T and the reduced temperature θ we have assumed $M = 29$ throughout ignoring the slight differences of the values of M for air, unburnt gas, and burnt gas at different temperatures.

The particular expression for E as function of θ was obtained by assuming the explosive to consist of a mixture of air and hydrocarbon such that the numbers of carbon atoms, hydrogen atoms, oxygen molecules, and nitrogen molecules were in the ratio 1:1.78:1.45:5.50. In determining the energy expression the change in composition involving some dissociations was taken into account; the deviation of the expression for E from linearity in θ is mainly due to the changes of the specific heats. We should like to emphasize once more, that our results do not pretend to represent the behavior of a particular substance accurately; they are only intended to be typical. As a matter of fact, calculations have been performed previously under the assumption that air, unburnt, and burnt gas have the same constant specific heats with the ratio $\gamma = 1.334$, the liberated energy - g_b being approximately .75 kcal/gm. The results were essentially the same as in the case here reported; except that the values of the impulse are somewhat higher and that the value of the impulse for case DIII is during some time nearer to that for case DI rather than to that for case DII.

4. Detailed Description of the Processes. We proceed to describe the motion of the gases and the variation of pressure and temperature resulting from the various modes of detonation and combustion. The flow processes will be described in terms of "waves".

Detonation, combustion, and shock waves are nothing but moving detonation, combustion, and shock fronts implying a discontinuity. Rarefaction and compression waves are moving zones across which pressure, temperature, and velocity vary. That such a description is suitable for actually occurring processes may be seen from Schlieren-photographs made by Payman and his collaborators. (See [8]).

We first consider the process resulting from a constant volume detonation, mode DIII. After the detonation the exploded gas has instantaneously acquired a pressure $p = 9.36 \text{ atm.}$ and a temperature $T = 2800^\circ\text{K.}$ Since the pressure difference between 9.36 atm. and 1 atm. at the interface between exploded gas and air cannot be maintained, waves are sent out from this interface which produce an adjustment. A shock wave travels into the quiet air raising its pressure to 5.91 atm. , its temperature to 542°K and accelerating it to a velocity $u = 1.4c_0$; here $c_0 = 347\text{m/sec} = 1.14 \text{ ft/sec}$ is the sound speed in air of atmospheric pressure and 300°K temperature. A rarefaction wave travels backwards into the exploded gas, reducing the pressure to 5.91 atm. , the temperature to 2550°K and accelerating the exploded gases to a velocity $u = 1.4c_0$. The rarefaction wave, in contrast to a shock wave, has a finite width which increases proportional to the time it has traveled. This rarefaction wave will be reflected at the closed end; due to this reflection, which takes place in the time interval between $t = .36t_0$ and $t = .67t_0$, the pressure at the closed end is reduced from 9.36 atm. to 3.25 atm. and the temperature from 2800°K to 2300°K . The reflected rarefaction wave meets the interface between burnt gas and air at the time $t = .94t_0$. At this time, the interface has moved out to the distance $2.31 l_0$ from the closed end. The reflected rarefaction wave will be partly transmitted and partly reflected at the interface. The effect of this interaction is a considerable deceleration of the interface. In the time interval $t = .94t_0$ to $1.83t_0$, the velocity decreases from $1.4c_0$ to $.57c_0$.

At the same time the temperature is reduced to 2170°K in the burnt gas and to 423°K in the air. The rarefaction wave transmitted through the interface will catch up with the shock wave (beginning at $t = 2.1 t_0$ at a distance $5.5 l_0$ from the wall). The interaction between rarefaction wave and shock wave consists of a gradual diminishing and slowing down of the shock wave. This process is extremely slow and, probably, goes on indefinitely; the reflected wave resulting from this interaction is negligible. However, the rarefaction wave reflected from the interface will again be reflected at the closed end; the resulting reflected wave will again be partly reflected, partly transmitted at the interface, and so on. Three such reflections at the closed end were determined. The pressure and the temperature at the closed end will gradually decrease in the course of these reflections; the pressure there will probably approach atmospheric pressure asymptotically. This decrease of pressure is shown in Table 3.

The results here described are exhibited in the diagram, Table 7, in which the distance x from the closed end is plotted as abscissa and the time t as ordinate. The units are l_0 and $t_0 = l_0/c_0$ respectively. The shock is indicated by a heavy line, the interface by a dotted line. To plot rarefaction waves, we have made use of the fact that the end points of the wave zone move with sound velocity (relative to the gas), which of course is different at both end points. The region in the (x,t) -plane covered by a rarefaction wave is therefore bordered by lines which indicate the path a sound wave (i.e. an extremely weak shock or rarefaction wave) would describe. Such lines we shall refer to as "sound lines." It is then natural to describe a rarefaction wave by showing several sound lines in the region covered in addition to those bordering this region.

The flow just described was determined on the basis of the assumption that heat conduction and viscosity can be ignored (despite the fact that both play a role in the creation of a

shock). It appears to be rather severe to ignore heat conduction in the neighborhood of the interface between burnt gas and air in view of the fact that the difference of the temperatures on both sides is 2000°K at first and still over 1600°K after 3 time units.*

Next we describe the processes resulting from the mode DIV, involving pre-compression. For the case of the pre-compression ratio $V_0/V'_0 = 6.03$, the gas has acquired the pressure 12.06 atm. and the temperature 600°K ; after the explosion the pressure is 58.4 atm. and the temperature 2900° . In the subsequent pre-expansion leading to "state (1)" the volume is only increased from $V'_1 = .166 V_0$ to $V_1 = .198 V_0$ (see Figure 3); the pressure drops to 47.32 atm. and the temperature to 2800°K . Incidentally, this latter temperature is independent of the degree of pre-compression; the reason is that the value of the total energy $E = e + g$ in state (1) is equal to that in the original state (0) since the energy increase during pre-compression is balanced by the energy loss during pre-expansion and the total energy does not change in a constant volume detonation. When the state (1) is reached, at the "effective initial time," the piston is removed and a flow process ensues which is similar to that resulting from a constant volume detonation without pre-compression. The only difference is that the initial volume is smaller, $V_1 = .198 V_0$, the initial pressure $p_1 = 47.37$ atm. is higher, while the initial temperature is the same. In consequence of these differences the various reflections take place faster. We have determined three reflections at the closed end, the last being completed at $t = 5.54 t_0$; the pressure has dropped to 1.4 atm. at that time (see Table 8).

* Calculations made on the basis of the standard heat flow theory indicate, however, that after 3 time units the width of the region in which 80 percent of the temperature difference is attained is only of the order of magnitude of one cm.

In the extreme case of the compression ratio $V_0/V'_0=103.2$, the volume is $V'_0 = .0097 V_0$, the pressure is 515.8 atm., the temperature 1500°K . As a result of the explosion, the pressure rises to 1176 atm. and the temperature to 3420°K . Through pre-expansion, the volume increases to $V_1 = .0425 V_0 = (1/23.5)V_0$, the pressure drops to 200.15 atm. while the temperature again is 2800°K . The reflections take place very quickly. The first reflection at the wall is completed at $t = .22 t_0$. It decreases the pressure to 2.8 atm., the temperature to 1250°K . These values remain there until the time $t = .44 t_0$ when the next reflection begins. We have stopped our calculations there (see Table 9). The impulse as function of the time t up to $t = .44 t_0$, plotted in Table 1, already shows clearly that the gain in impulse is slight as compared to the tremendous increase in the pre-compression rate. (Rough calculations have indicated that the rate of increase in impulse as function of the pre-compression rate begins to drop when its value is near $V_0/V'_0 = 6$.)

The flow processes resulting when a detonation wave travels either away from the wall or toward the wall are slightly more complicated. In mode DI, see Table 10, a detonation wave travels away from the wall with the speed $6.7 c_0 = 2.14 \text{ km/sec}$. It raises the pressure to 17.96 atm. and the temperature to 2980°K . The detonation wave is immediately followed by a rarefaction wave through which the pressure is lowered to 4.18 atm. and the temperature to 2400° . The width of the rarefaction zone increases proportionally with the time since the beginning of the process. Initially, therefore, this drop is instantaneous,* hence, at the closed end, only the pressure 4.18 atm. and the temperature 2400° are observed. When the detonation wave has reached the interface

* The statement is correct only inasmuch as the assumption is correct that the detonation is a sharp discontinuity. If the finite width of the reaction zone in a detonation were taken into account a continuous process would be found.

chemical reaction stops; a shock wave proceeds into the quiet air and a shock wave is sent back into the burnt gas. The shock in air raises the pressure to 18.9 atm. and the velocity to $3.16 c_0 = 1.10 \text{ km/sec}$; its own speed is $4.04 c_0 = 1.41 \text{ km/sec}$. The shock wave sent back into the burnt gas is extremely weak. After having traversed the rarefaction wave following the detonation front, this shock raises the pressure from 4.18 to 4.64 atm. and the temperature from 2400° to 2430° . The same happens at the closed end as seen from the diagram on Table 10. The rarefaction wave that followed the detonation is traversed by the reflected shock just mentioned and then meets the interface between burnt gas and air; it is partly transmitted through the interface partly reflected. This reflected rarefaction wave is again reflected at the closed end, thus decreasing the pressure from 4.64 to 1.93 atm. and the temperature from 2430° to 2070° . This reflection process takes place between $t = .61 t_0$ and $1.61 t_0$. Pressure and temperature at the closed end then remain constant up to $t = 2.42 t_0$, when the next reflection arrives.

In Mode DII, see Table 11, the detonation wave travels from the interface towards the closed end. Its speed is again $6.17 c_0 = 2.14 \text{ km/sec}$ and it raises pressure and temperature to 17.96 atm. and 2980°K . A shock moves into the quiet air raising the pressure to 2.84 atm. and accelerating the air to a velocity of $.815 c_0$. Consequently, the rarefaction wave which follows the detonation is stronger than that occurring in case DI, since it has to accelerate the burnt gas to the velocity $.815 c_0$ and to cause the pressure to drop from 17.96 atm. to 2.84 atm. When the detonation arrives at the closed end, at $t = .16 t_0$, it is reflected as a shock through which the pressure rises from 17.96 atm. to 60.90 atm. and the temperature from 2980° to 3675°K . The rarefaction wave following the detonation is traversed by the reflected shock and then is reflected at the closed end. During this reflection process which takes place

from $t = .16 t_0$ to $t = .74 t_0$, the pressure at the closed end drops from 60.90 atm. (see Table 3) to 2.2 atm. and the temperature from 3675°K to 2210°K . The shock and the rarefaction wave, reflected from the closed end, will eventually meet the interface between air and burnt gas. These waves will be partly reflected, partly transmitted. The reflected shock is very weak; it arrives at the closed end at the time $t = 1.125 t_0$ and is then reflected. The pressure at the closed end has the constant value 2.2 atm. during the time interval from $t = .74 t_0$ to $t = 1.125 t_0$. Due to the reflection of the shock wave coming in at $t = 1.125 t_0$, the pressure rises suddenly to the value 4.8 atm., but then drops quickly because of the rarefaction wave which follows the shock wave. (This pressure drop was only estimated; it is indicated by a dotted line in Tables 1 and 3).

We now proceed to describe in detail the modes CI and CII involving combustion fronts with various effective burning speeds.

In mode CI combustion starts at the closed end and proceeds into the unburnt gas. The combustion produces a compression wave pre-compressing the burnt gas (see supplement [5] to the Manual). Since the burning speed c_f is assumed to be constant, the pre-compression wave produces a constant compression and hence is a shock wave. This shock wave turns out to be very weak. Nevertheless it accelerates the gases to a velocity u much greater than the burning speed c_f . Consequently, the speed U_f of the flame front relative to the tube

$$U_f = u + c_f$$

is also much larger than the burning speed c_f . The relationship between the normal burning speed c_f and the flame speed U_f will be seen from the Table in p. 15. We have in particular

$$U_f = 27.7, \quad 125. \quad , \quad 336. \quad , \quad \text{m/sec} .$$

Thus, for a burning velocity c_f of 3.5 m/sec. which may be a possible value for a proper plane combustion front, we find

a flame speed $U_f = 28$ m/sec. The flame speed $U_f = 336$ m/sec or .97 sound speed is rather high. Diagrams for the three cases of mode CI are shown in Tables 12, 13, and 14.

The pressure increases through the pre-compression shock to the values $1.10 p_0$, $1.53 p_0$, $2.88 p_0$ respectively. The combustion process involves only a slight pressure drop; it does not deviate much from a constant pressure burning. Therefore the pressure at the closed end is still greater than 1 atm. as indicated in Table 5. For the cases $c_f/c_0 = .01$, $.05$, and $.15$ we find $p/p_0 = 1.10$, 1.50 , 2.52 . Since the combustion process is slow the impulse keeps increasing for a long time; the longer the time, the smaller its rate of change (the excess-pressure). It is not obvious in which case the greatest final value of the impulse is attained, but during the intervals of time considered, greater burning speed yields greater impulse. In the case $c_f/c_0 = .15$ the pressure of 2.52 atm. remains at the closed end up to the time $t = 3.6 t_0$; the impulse has reached the value $I = 5.5I_0$ at that time, see Table 1.

In the mode CII in which the flame starts at the interface between combustible gas and air, a shock wave is sent into the air accelerating in to velocities $u = .03 c_0$, $.16 c_0$, $.42 c_0$ respectively; see the diagrams in tables 12, 13, and 14. The interface between compressed air and burnt gas then moves with the same velocity in a direction away from the closed end. Consequently, the flame speed U_f relative to the tube is smaller than in mode CI, viz $U_f = 15.2$, 76.5 , 250 m/sec as seen from the Table on p. 15. The pre-compression shock which proceeds into the unburnt gas is weaker than the pre-compression shock for mode CI. It raises the pressure only to $p = 1.05 p_0$, $p = 1.27 p_0$, $p = 2.10 p_0$ respectively. The pressure at the closed end, however, is greater than in the mode CI because of the reflection of the pre-compression shock. The pressures there are $1.10 p_0$, $1.60 p_0$, $4.08 p_0$ as shown in Table 5. It is interesting to see

that the pressure at the closed end for mode CII is only slightly greater than for mode CI for c_f up to $.05 c_o$. In other words the shock reflection at the closed end essentially merely balances the weakening effect due to the exhaust motion of the burnt gas and compressed air; only for burning velocities c_f greater than $.05 c_o$ will the effect of the reflection become noticeably dominant. The most remarkable feature is that in the case of the high burning speed $c_f = 15 c_o = 52.05$ m/sec the impulse for mode CII exceeds that for the detonation mode DI from about $t = 2 t_o$ on.

The flow resulting in this case is shown in Table 14. The shock wave reflected from the closed end meets the flame front at $t = 1.01 t_o$ and retards it. In addition to the transmitted shock wave, which is very fast, a weak rarefaction wave is sent back to the closed end arriving there at $t = 1.23 t_o$. The pressure at the closed end begins to drop slowly from that time on. A complicated wave pattern ensues which involves weak rarefaction waves and shock waves being reflected at the closed end and entails slight variations of the pressure there. When the flame has reached the closed end, which is estimated to take place at about $t = 3 t_o$, a strong rarefaction wave will start at the closed end and the pressure will drop considerably. It is interesting to note that the wave pattern here described is rather well confirmed experimentally by Payman's Schlieren photographs, see Payman and Titman, [8], p. 444, Fig. 4.

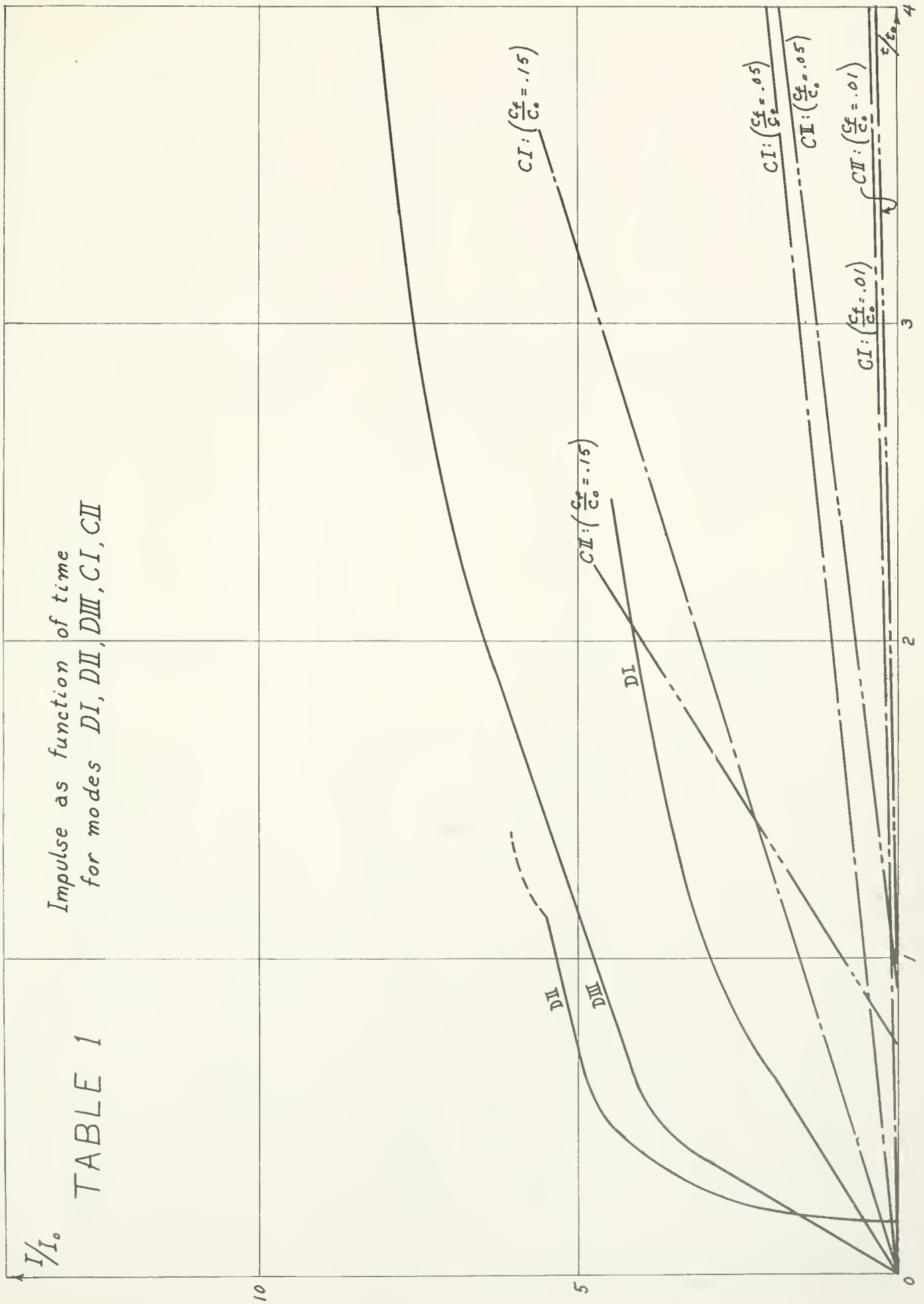
The discussion of the last case of mode CII shows particularly clearly that our way of attack will in general be successful only for the first stages of the process. If the information for a wider range of time is desired, different methods, probably methods of sonic approximation, would have to be employed. Since, however, the first part of the process yields the main part of the impulse, our investigations appear to give a significant contribution to the discussion of the effectiveness of certain jet operations.

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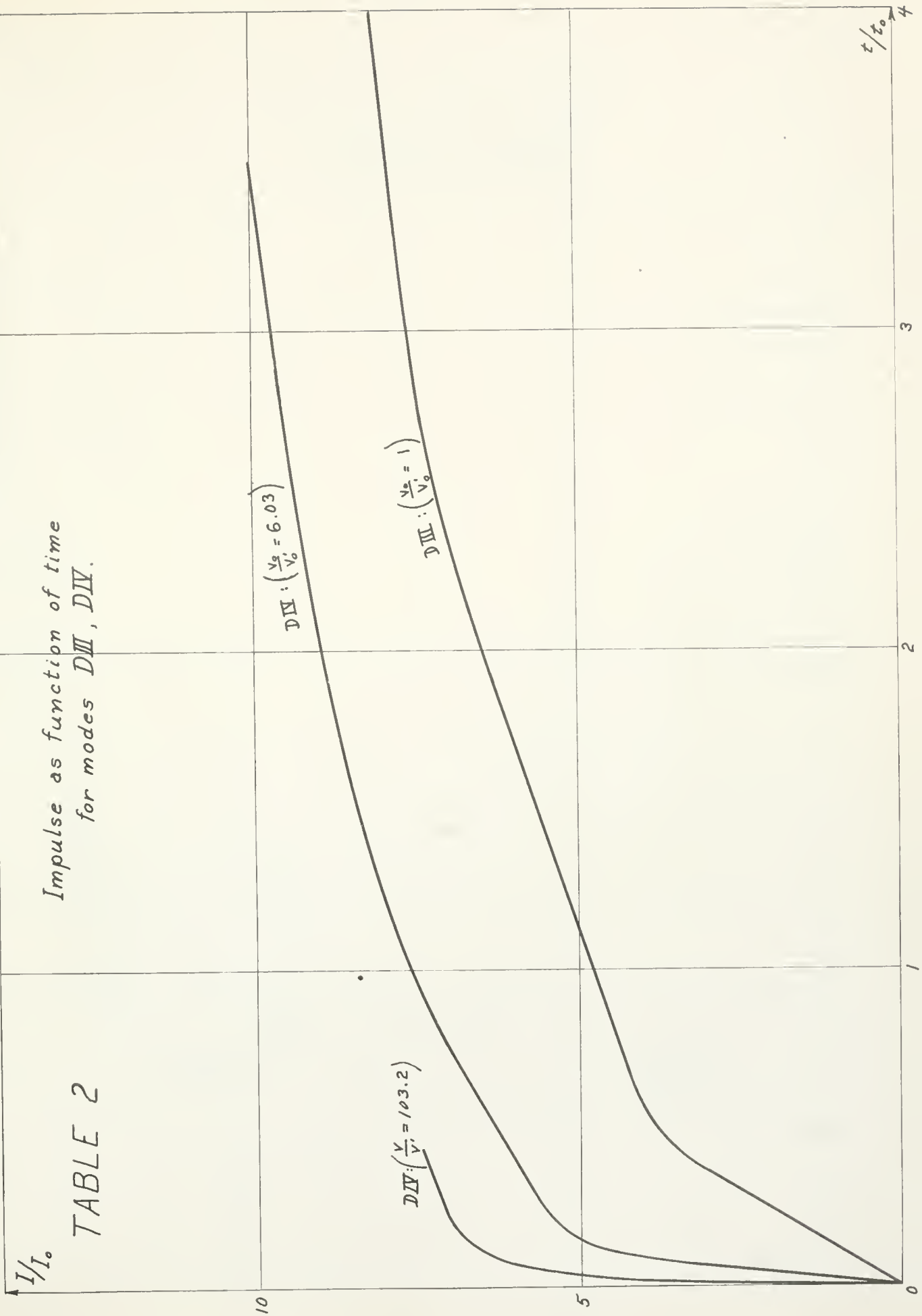
Impulse as function of time
for modes DI, DII, DIII, CI, CII

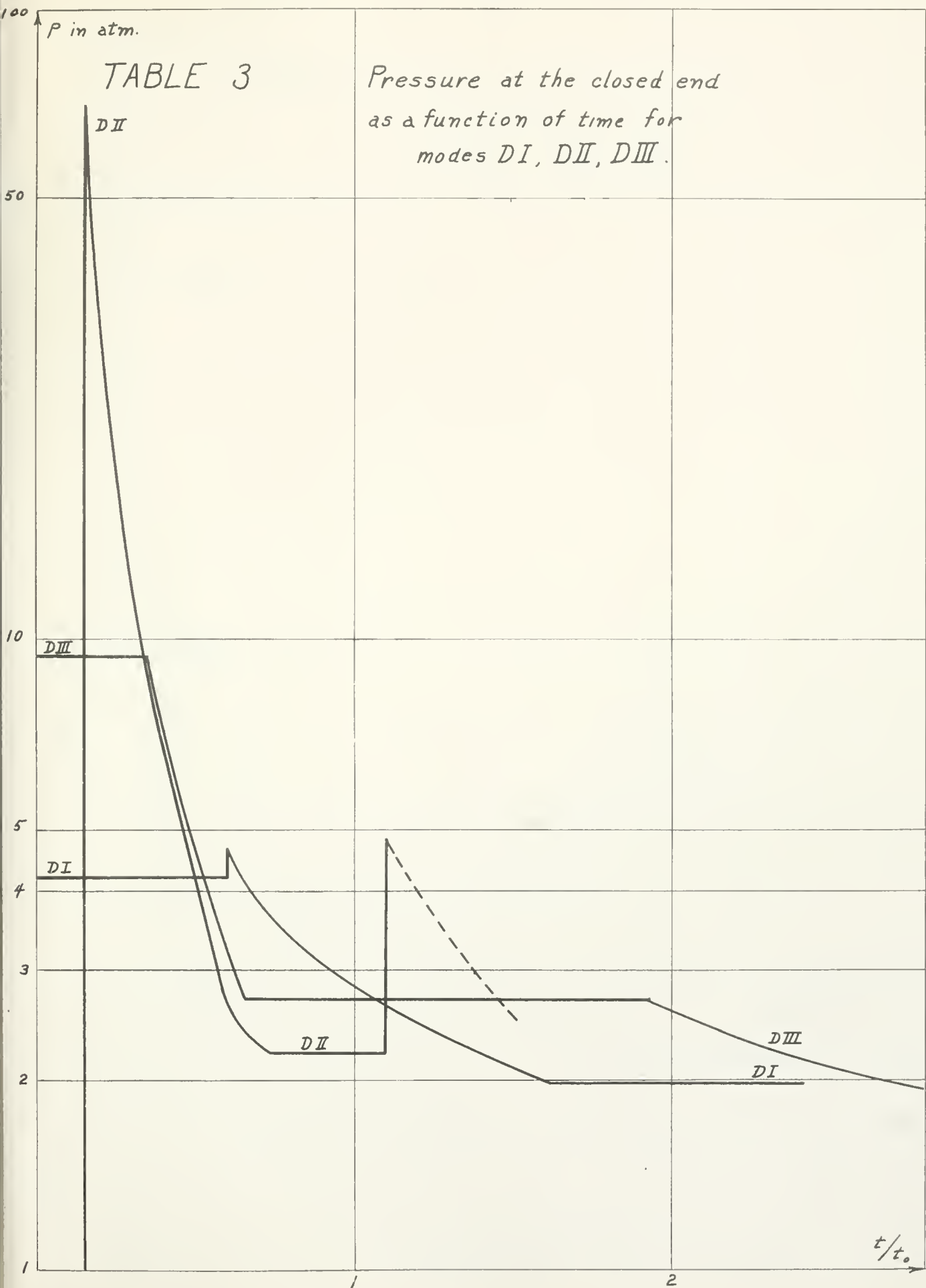
TABLE 1



Impulse as function of time
for modes DIII, DIV.

TABLE 2





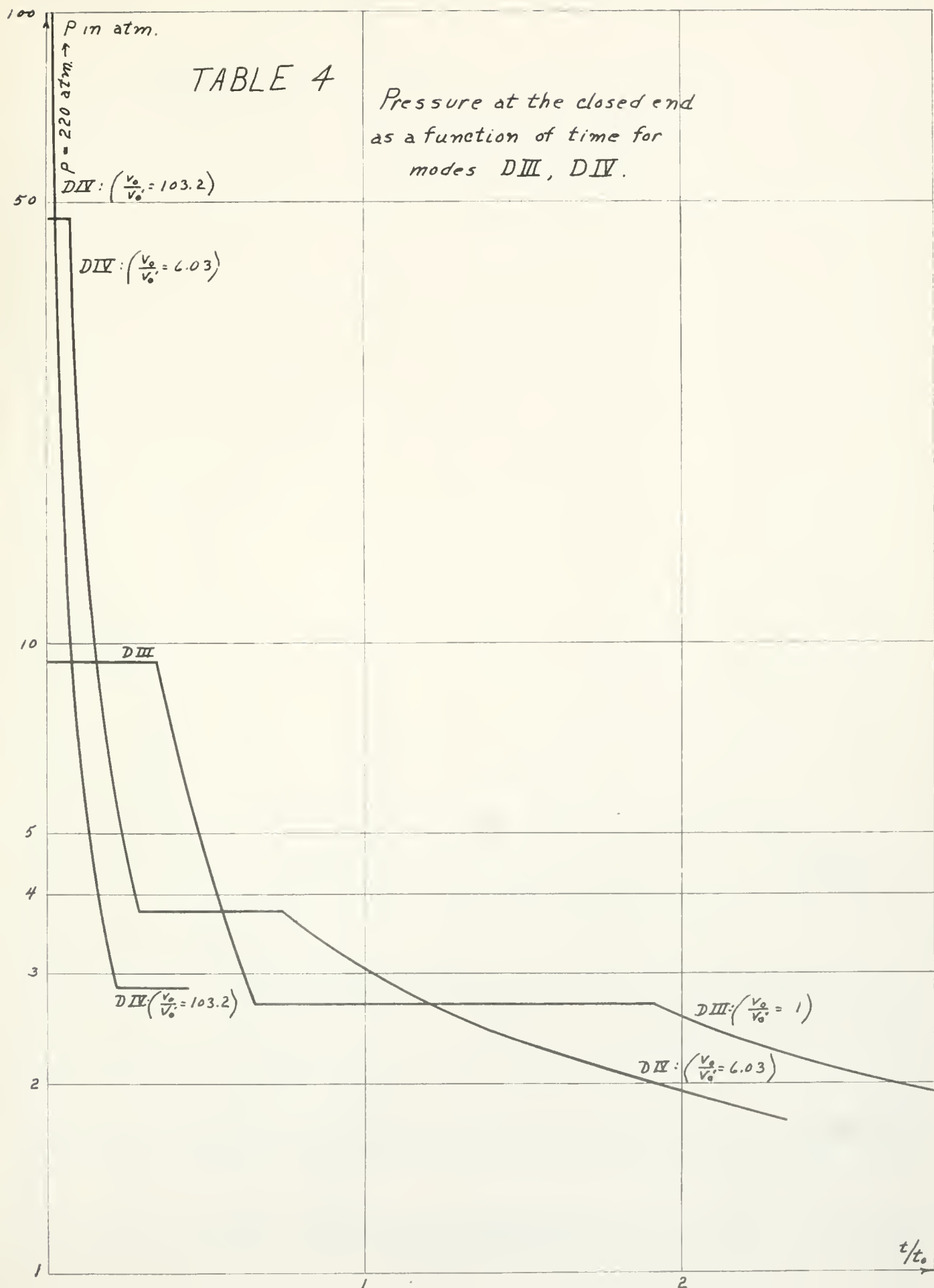


TABLE 5

Pressure at the closed end
as a function of time for
modes DIII, CI, CII.

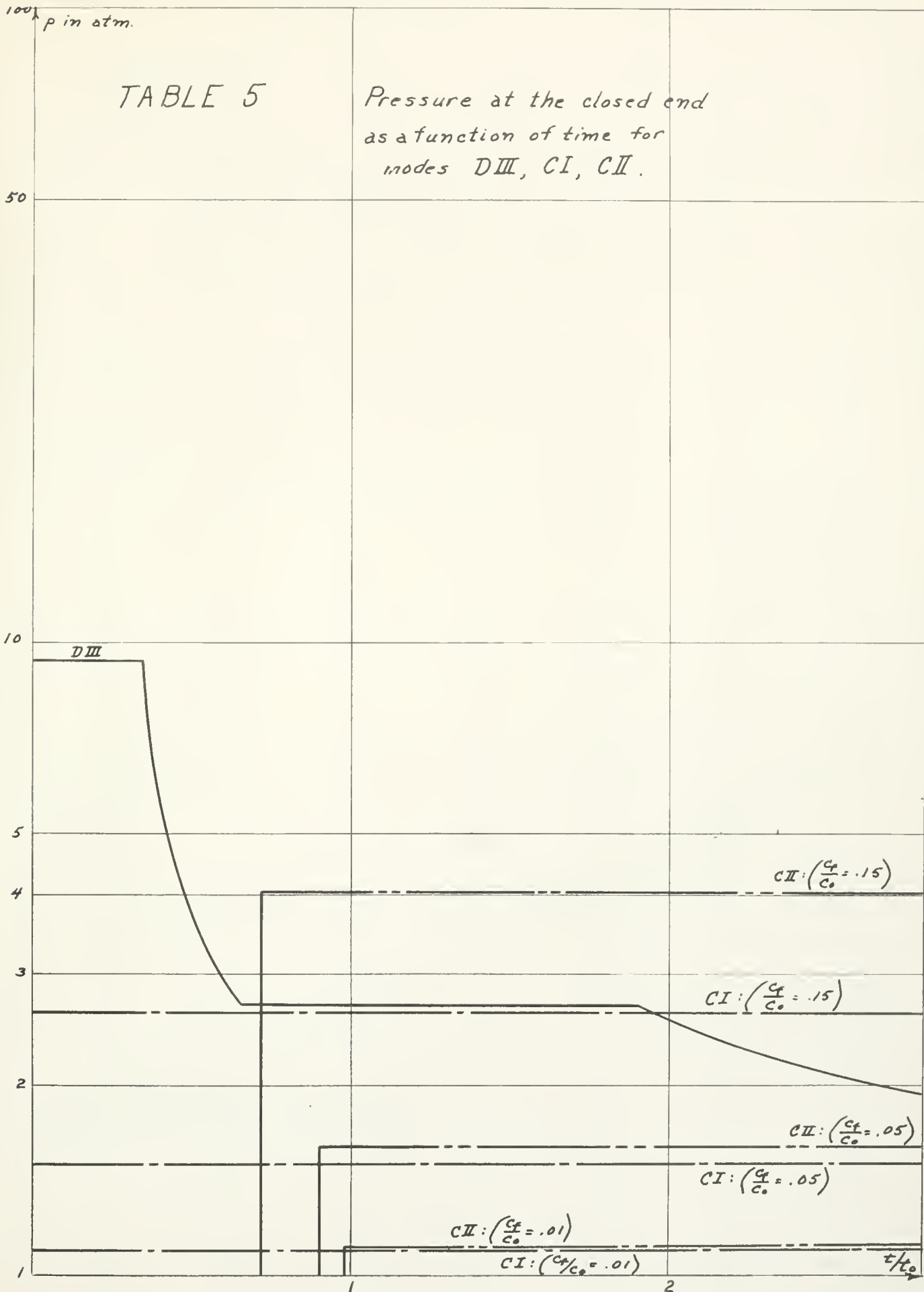
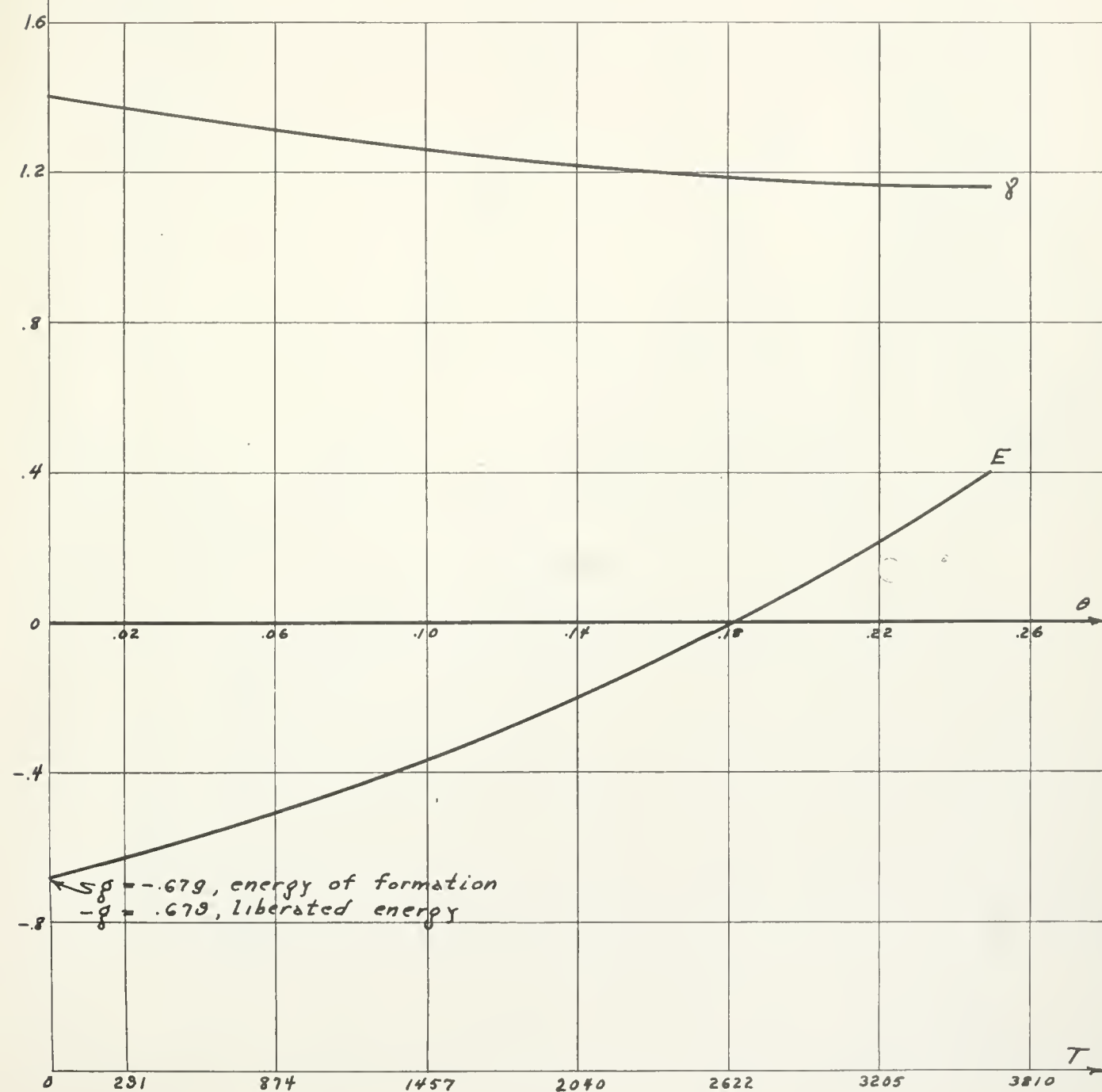


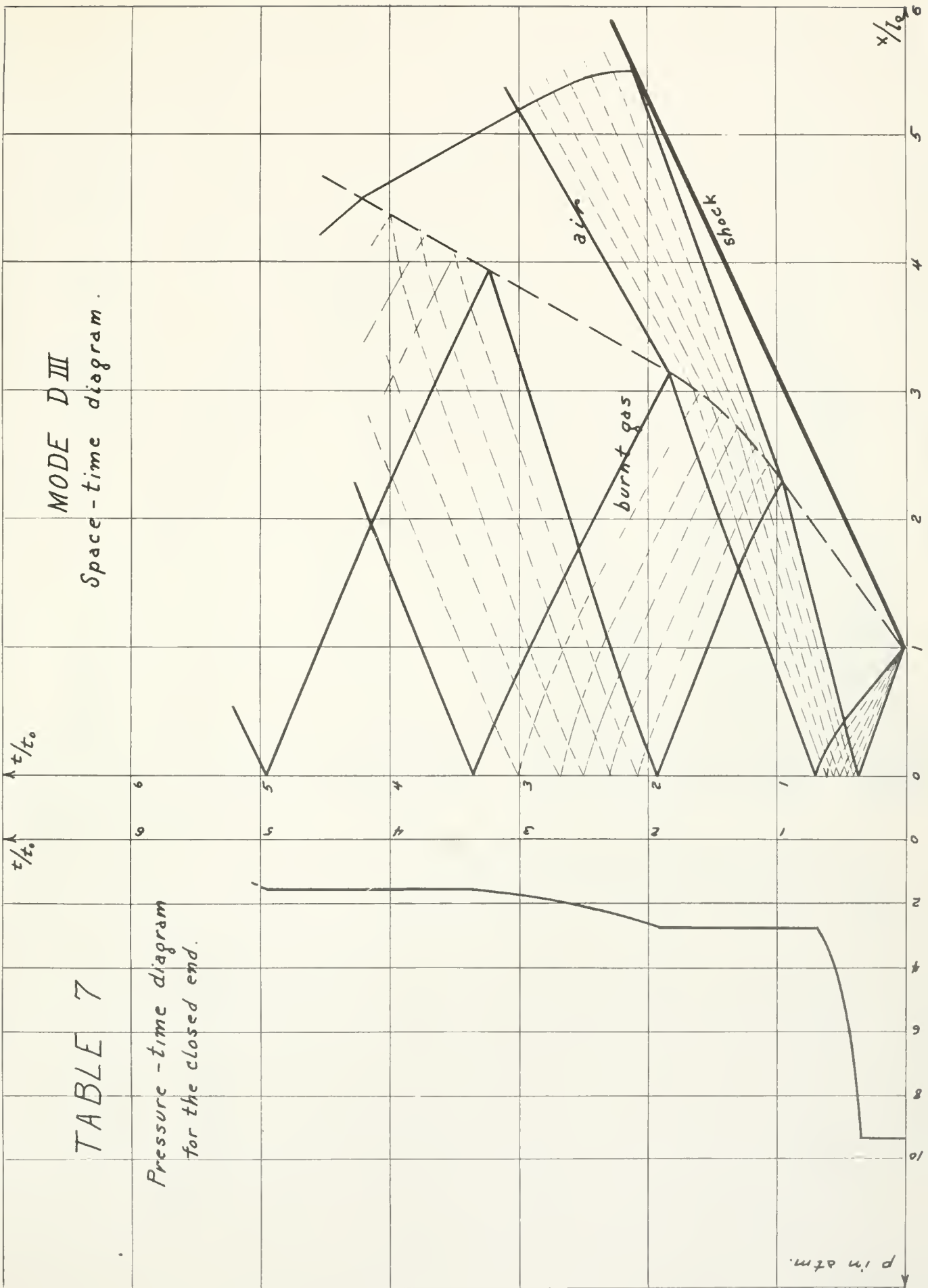
TABLE 6

Total energy per unit mass, $E = c + g$ (in $\frac{\text{k.cal.}}{\text{gr.}}$),
and ratio of specific heats γ as functions of
the temperature T (in $^{\circ}\text{K}$), or the reduced
temperature $\theta = pr$ (in $\frac{\text{k.cal.}}{\text{gr.}}$) for the burnt gas.



MODE D III

Space-time diagram.



MODE DIV : $\left(\frac{V_0}{V_1} = 6.03\right)$
Space - time diagram.

TABLE 8

Pressure - time diagram
for the closed end.

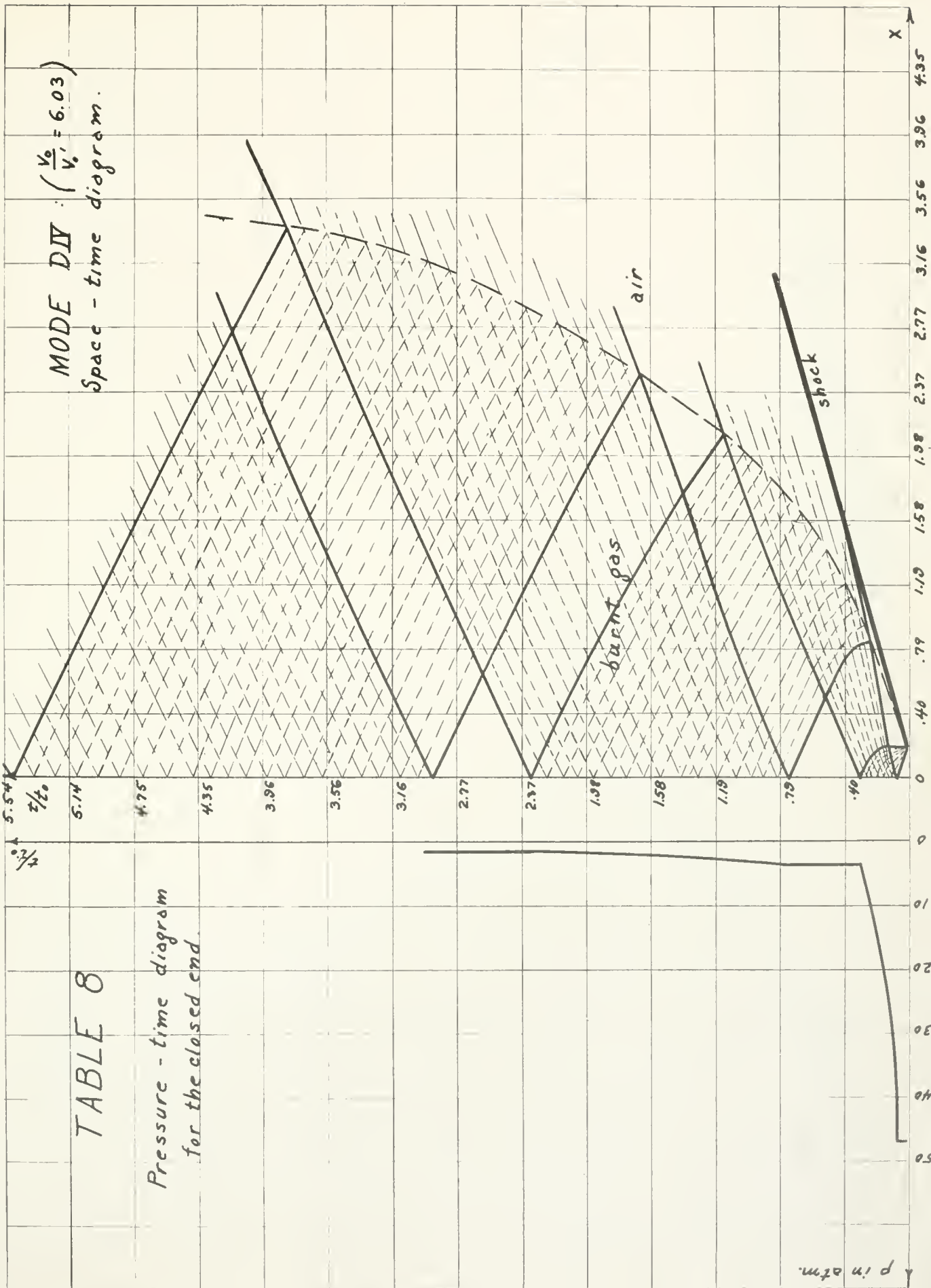
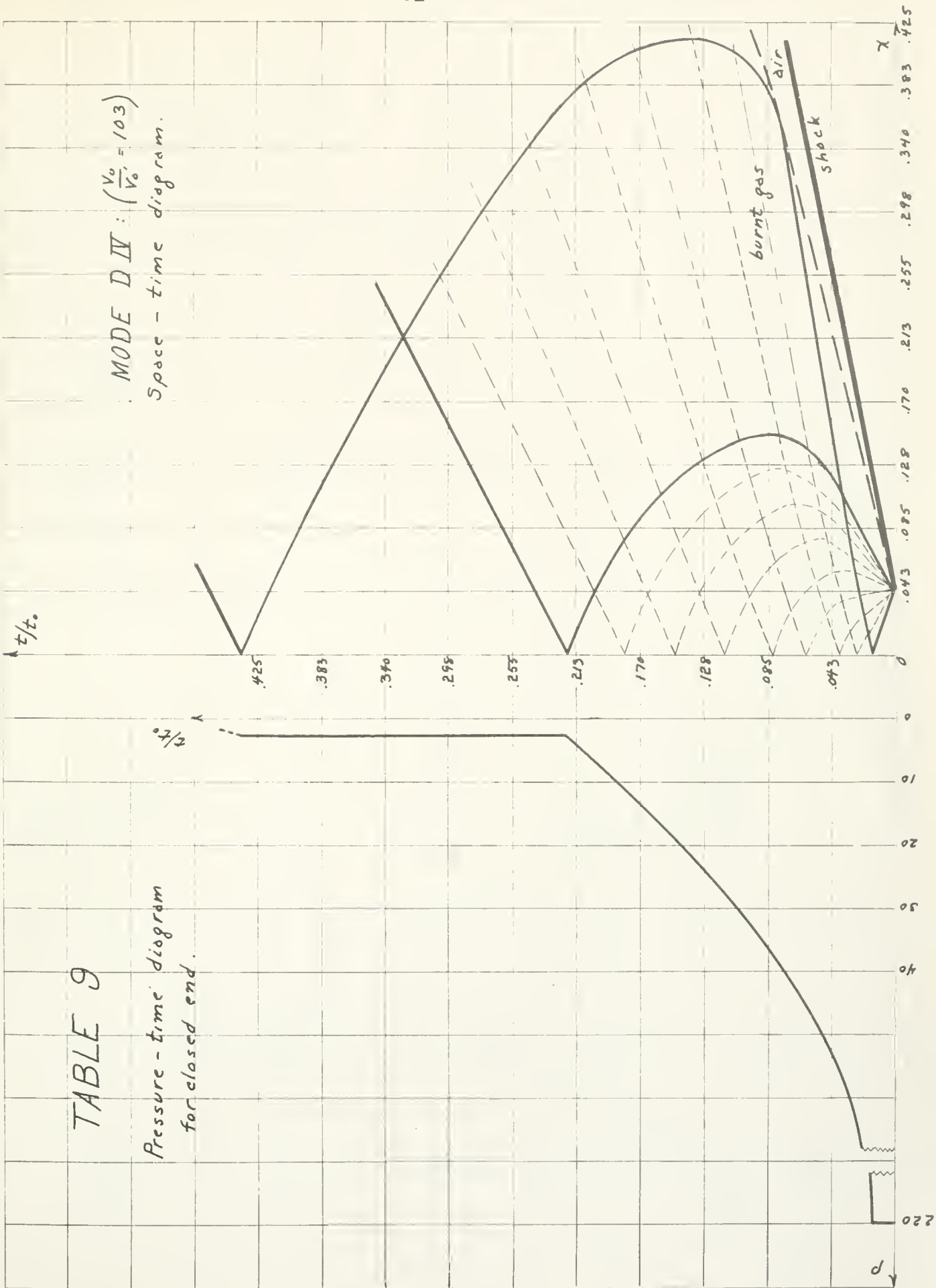


TABLE 9

Pressure-time diagram
for closed end.

MODE D IV : $\left(\frac{V_0}{V_0'} = 103\right)$
Space-time diagram.



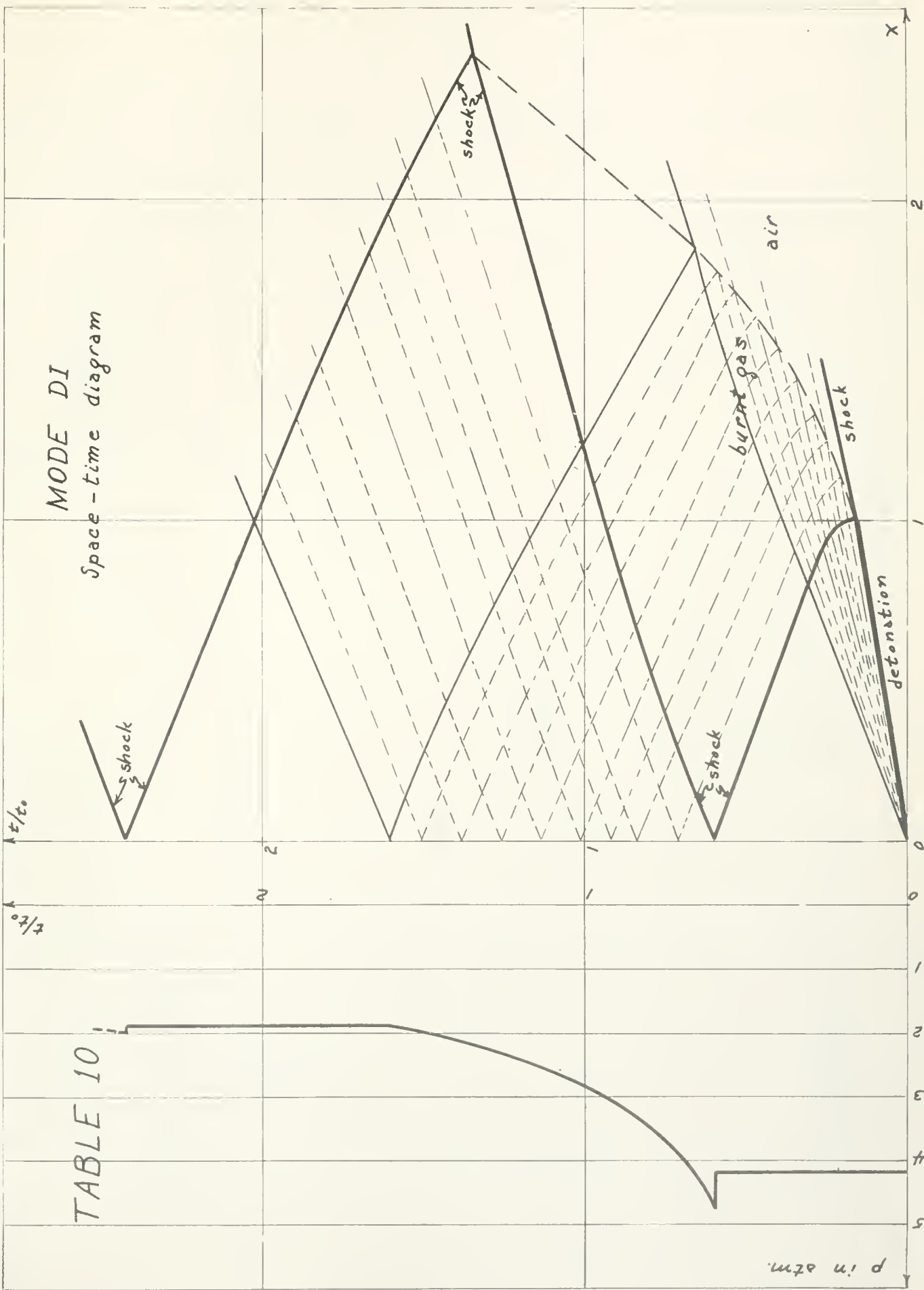
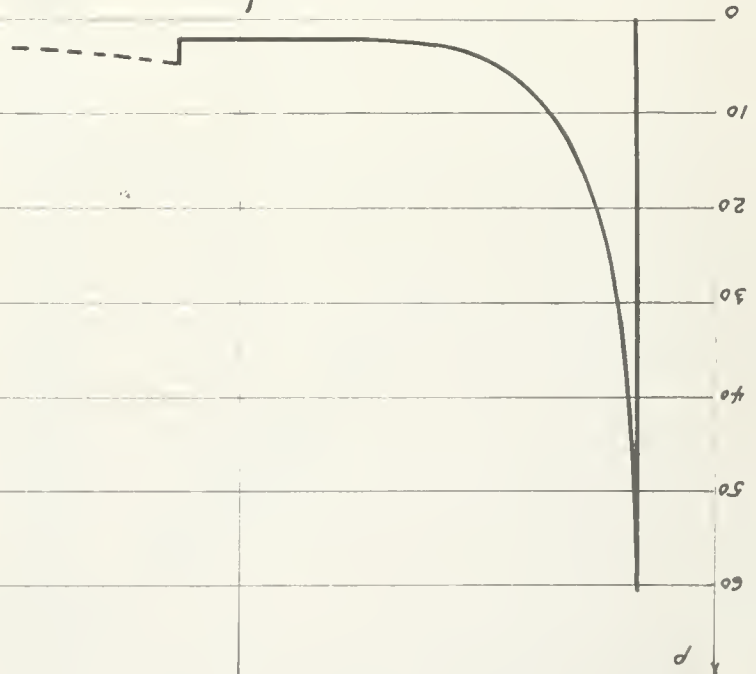
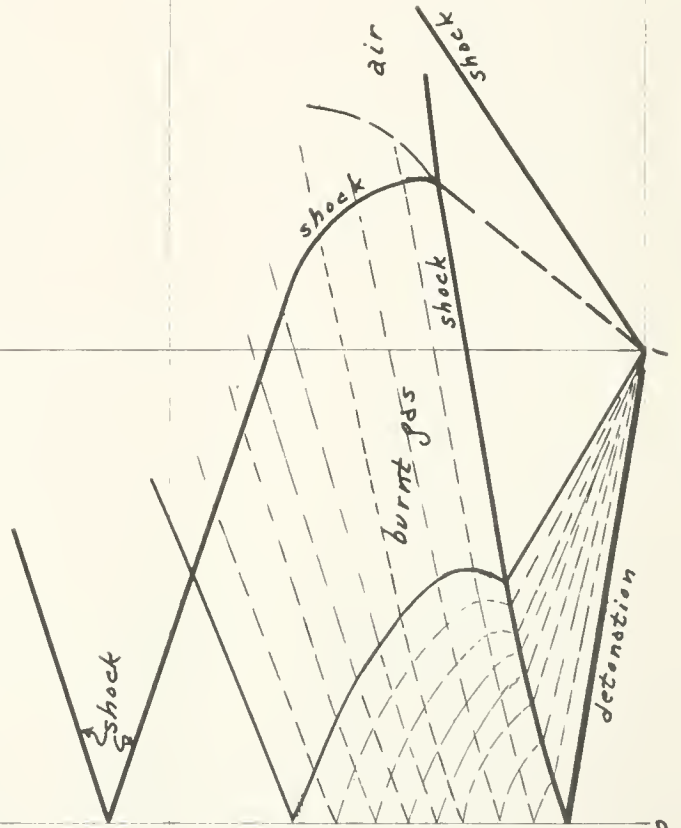


TABLE II

Pressure - time diagram
for the closed end

 t/t_0


MODE DII
Space - time diagram.

 t/t_0

 x

2

0

0

10

20

30

40

50

60

TABLE 12

MODES CI AND CII

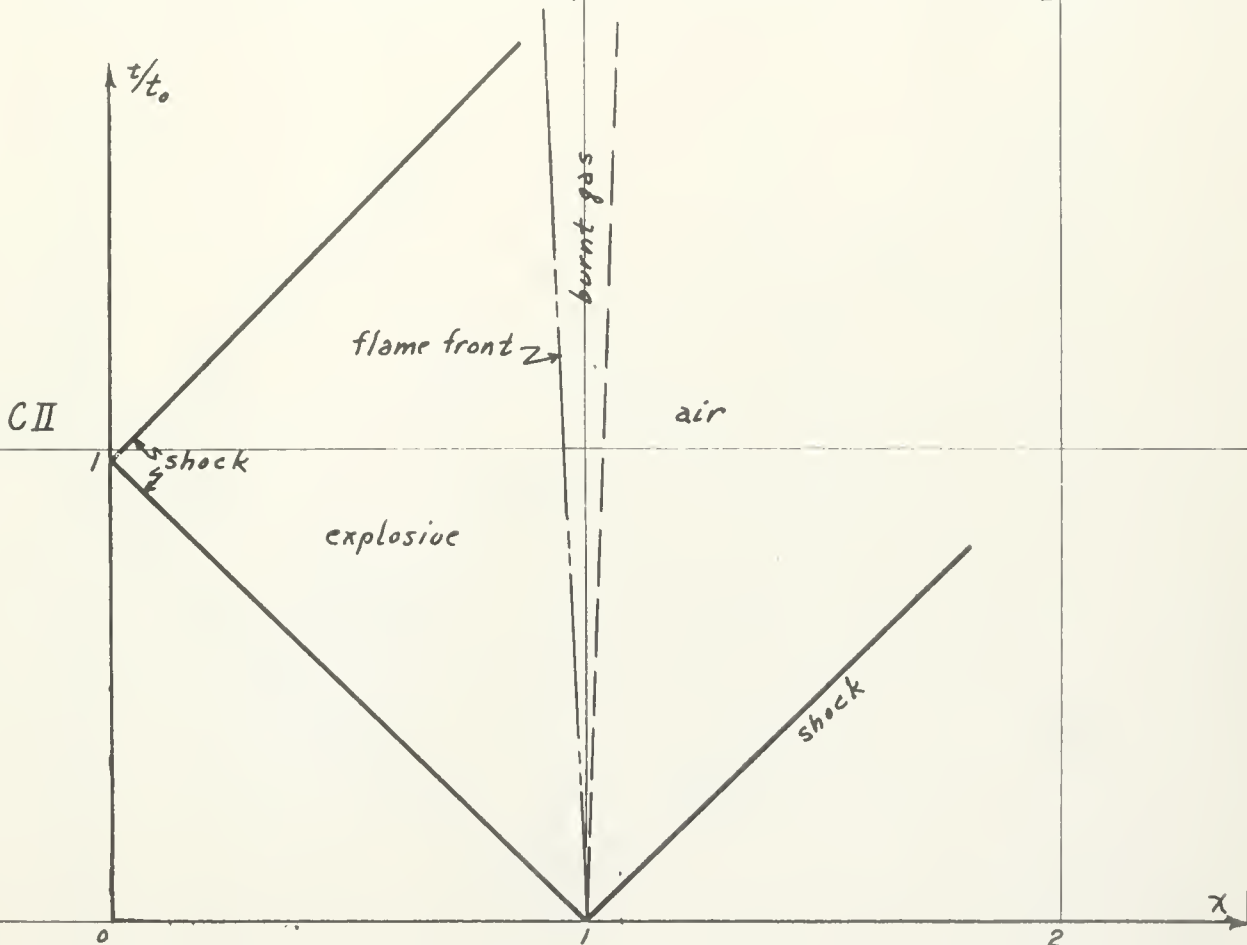
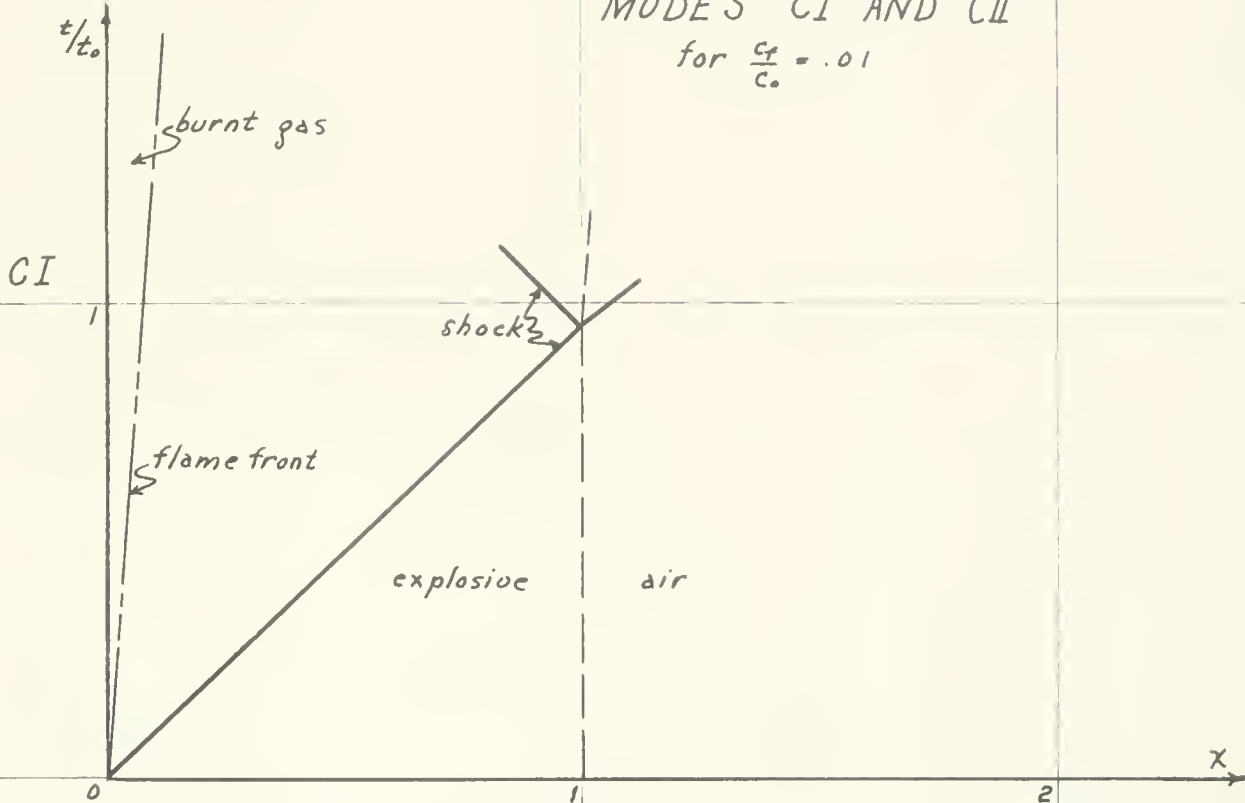
for $\frac{c_f}{c_o} = .01$ 

TABLE 13

MODES CI AND CII
for $\frac{C_T}{C_0} = .05$

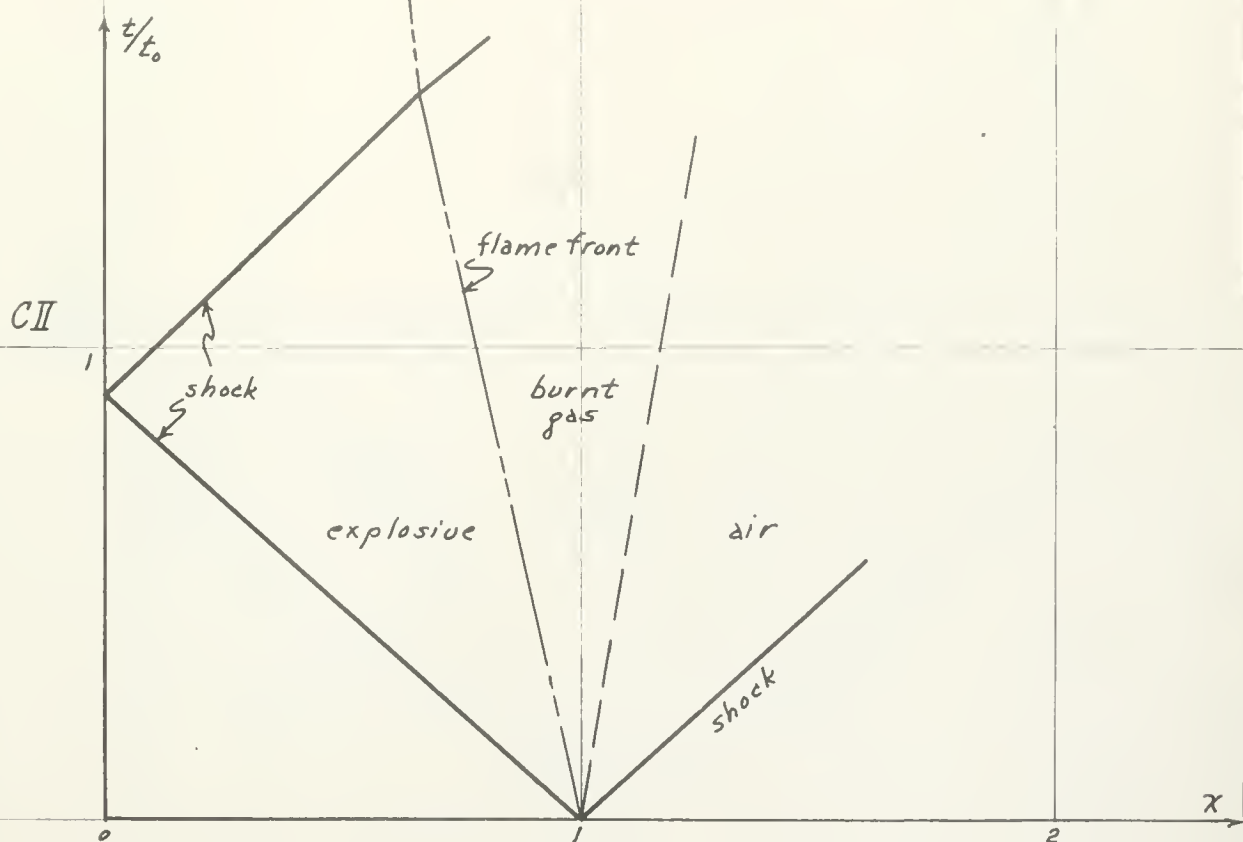
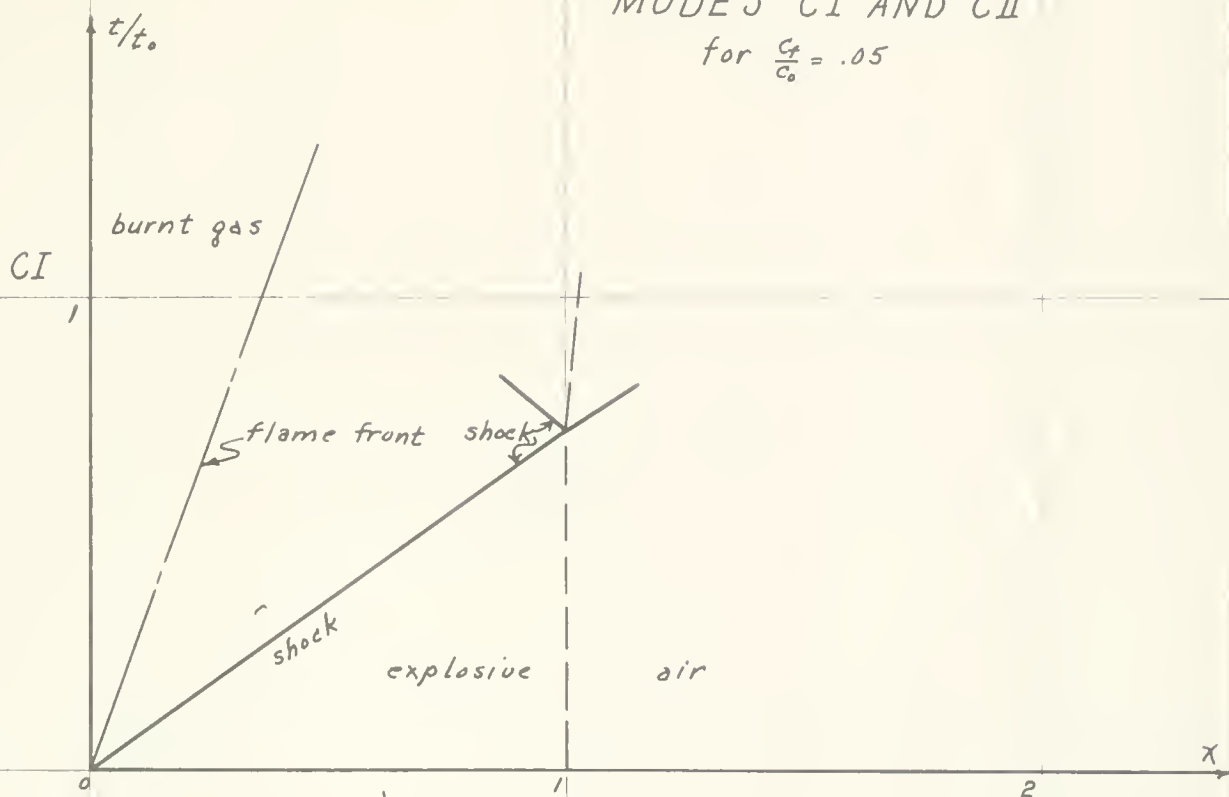
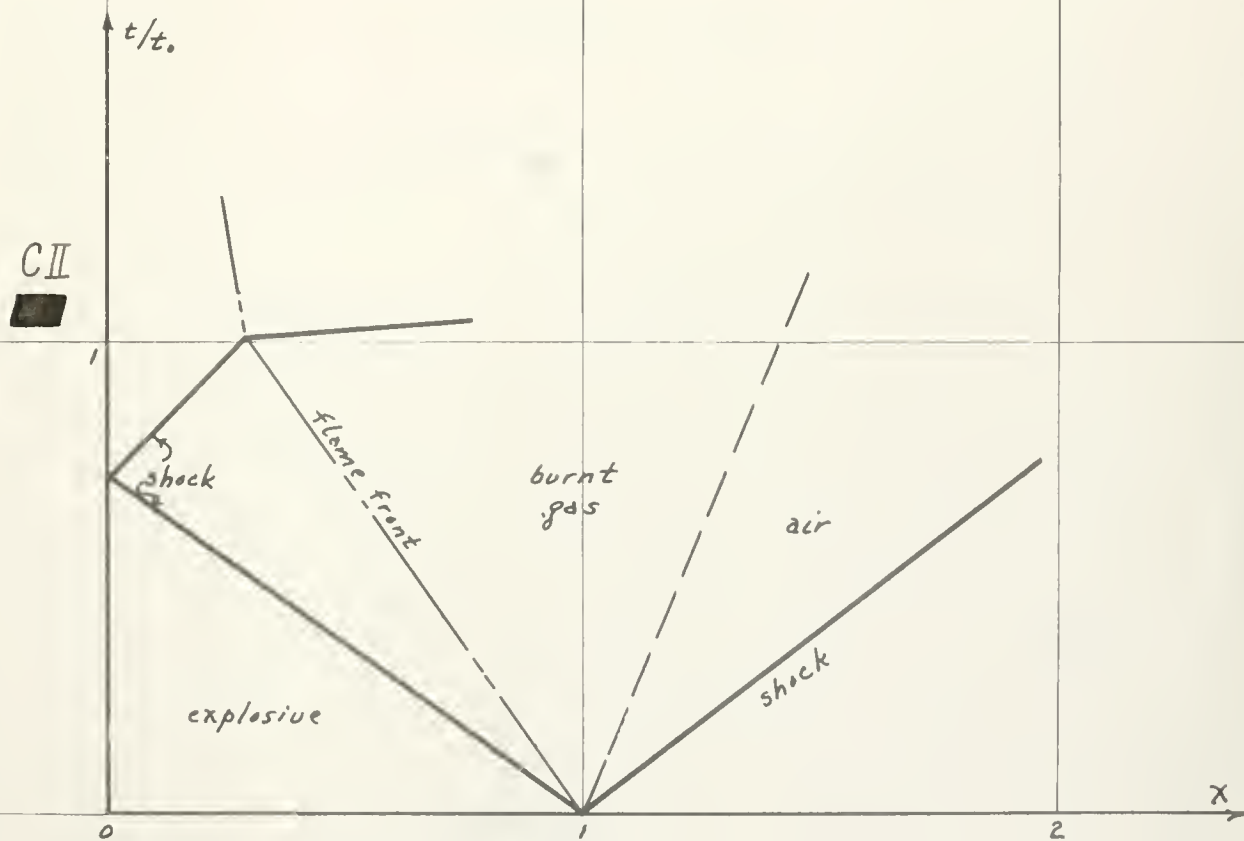
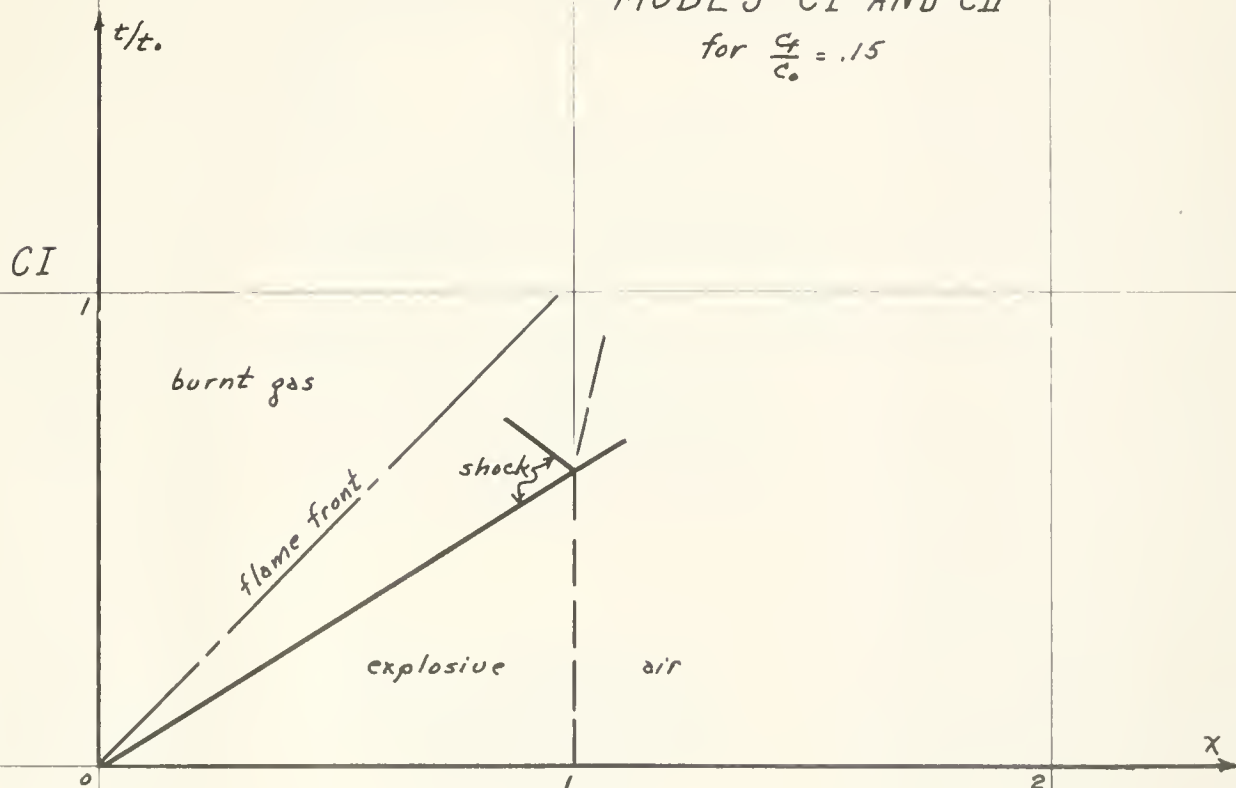


TABLE 14

MODES CI AND CII

for $\frac{C_f}{C_0} = .15$ 

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AUTHOR
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